On the Heat Transmission from a Rotating Cylinder of Infinite Length.

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1. Introduction.

The heat transmission from a rotating circular cylinder immersed in an air stream is a very interesting problem about the free and forced convections. In gas turbines, air-preheaters and other mechanical or chemical industries, it seems practically most important for the heat exchanger problems.

These problems, however, are very difficult to be solved theoretically and experimentally; they have many factors not to be neglected carelessly, and the various results have been introduced by many researchers. [1][2][4]

In this paper, after introducing the simple solution of temperature distribution for a potential flow with circulation, the writer tried to have the empirical formulae for mean Nusselt's number Num from the experimental data.

2 .Theoretical Analysis.

Denoting the density, specific heat and thermal conductivity of air stream by ρ, C_p and λ respectively, the temperature distribution at any point outside the cylinder can be represented as

$$\rho Cp\frac{\partial\Theta}{\partial t}+\rho Cp\nabla(W;\Theta)=\varepsilon+\lambda\nabla^2\Theta \qquad (1),$$

where, ε is the dissipation factor of heat.

Now, neglecting the pressure drop and viscosity; i.e, treating as potential flow, eq. (1) becomes

$$u\frac{\partial\Theta}{\partial x}+v\frac{\partial\Theta}{\partial y}=a\left(\frac{\partial^2\Theta}{\partial x^2}+\frac{\partial^2\Theta}{\partial y^2}\right) \qquad (2).$$

Using the equation of continuity and irrotational condition, and further, introducing the potential and stream functions ϕ, Ψ, we get finally

$$\frac{\partial\Theta}{\partial\phi}=a\left(\frac{\partial^2\Theta}{\partial\phi^2}+\frac{\partial^2\Theta}{\partial\Psi^2}\right) \qquad (3).$$

The boundary conditions now considered are as follows :

$$\Theta=\Theta_0 \quad \text{for} \quad r=R$$
$$\text{and} \quad \Theta=0 \quad \text{for} \quad r\to\infty \qquad (4).$$

Representing the solution of eq. (3) by Fourier's integral and using the conditions (4), the temperature distribution at any point is finally represented by

$$\Theta(\phi,\Psi)=\frac{\Theta_0}{\pi}e^{h\phi}\int_0^\infty exp.[\sqrt{\alpha^2+h^2}\,\Psi_R-\sqrt{\alpha^2+h^2}\,\Psi]\times$$

$$\frac{h\cos\alpha\phi+\alpha\sin\alpha\phi}{\alpha^2+h^2}d\alpha \qquad (5),$$

where, $h=1/2a=\rho C\phi/2\lambda$ [2], and ϕ takes always positive values.

The heat quantity transmitted from the cylinder surface per unit time is

$$dq=-\lambda\left[\frac{\partial\Theta}{\partial\phi}\delta\Psi+\frac{\partial\Theta}{\partial\Psi}\delta\phi\right]_{r=R},$$

and if we take the heat transmission coefficient $\alpha_m$kcal/m²-h-⁰C, the following equation is hold at the same time.

$$dq=\alpha_m ds\,\Theta_0$$

Therefore, we have the mean α_m all over the surfaces

$$\alpha_m=\frac{\lambda h}{\pi^2 d}\oint e^{h\phi_R}[K_0(h\phi_R)+K_1(h\phi_R)]d\phi_R \qquad (6),$$

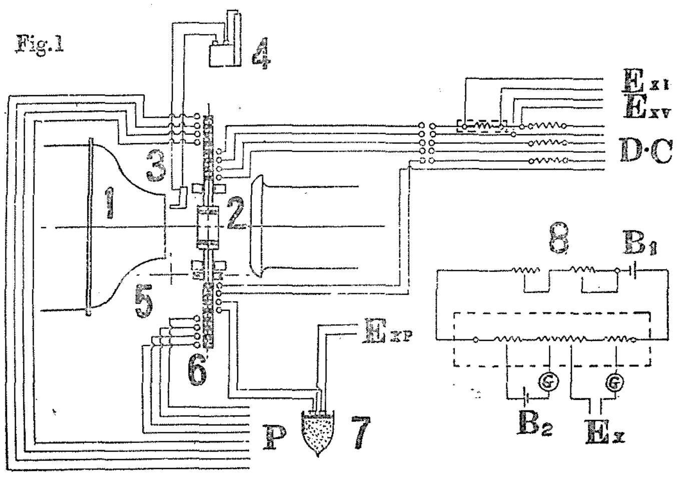
and introducing the non-dimensional coefficient Num

$$\mathrm{Num}=\frac{\alpha_m d}{\lambda}=\frac{h}{\pi^2}\oint e^{h\phi_R}[K_0(h\phi_R)+K_1(h\phi_R)]d\phi_R \qquad (7),$$

where, $\oint$ means the integral referred to all over the surfaces and K_0, K_1 are the modified Bessel Functions of the second kind.

3. Experiment.

Fig.1



Using a 400mm diameter Göttingen type wind tunnel 1, as showm in Fig. 1, air stream velocity v_∞ is changed into the range between 2m/s, and 18m/s, and as for the peripheral velocity v_θ of cylinder 2, it was changed stepwise as 1.83, 3.07 and 5.26 m/s respectively by the motor (¼HP)5.

To avoid the end effects, the guide cylinders which bounded by the bakerite rings are installed at both ends of the main cylinder ($d=40$mmϕ) and heated with the Ni-Cr wires (0.5mm dia.).

The surface temperatures are measured with the Cu-Constantan thermo-couples, each diameter of which is 0.15mm and at five points on cylinders the temperature is measured.

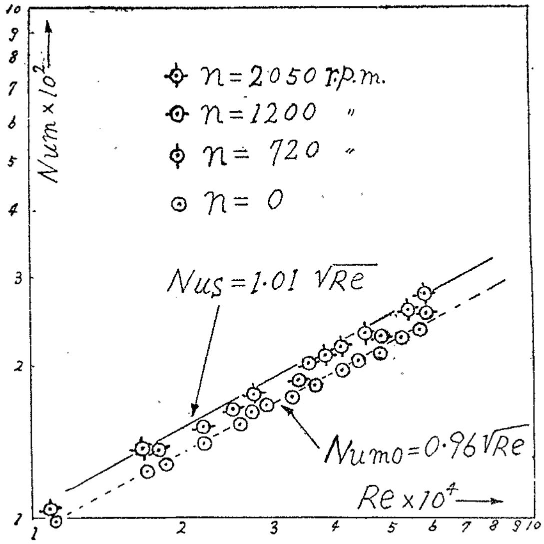
If we measure the heat quantity Q^{kcal}/h necessary to heat the cylinder and temperature difference $\triangle\vartheta^0 c=(\vartheta_0-\vartheta_\infty)$ ⁰C at the cylinder surface, the heat transmission coefficient α_m kcal/m²-h-⁰C can be determined by the following eqution.

$$Q=0.8601\ V\ I=\alpha_m F(\vartheta_0-\vartheta_\infty) \tag{8}$$

where, F represents the surface area [m^2] of main cylinder.

We verified[(3)] in the cases of free and forced convections that the heat transmission coefficient α_m Kcal/m^2-h-^{0}C are indifferent from the temperature difference $\Delta\vartheta^0 C$, and from these data we constructed the Figs. 2 and 3.

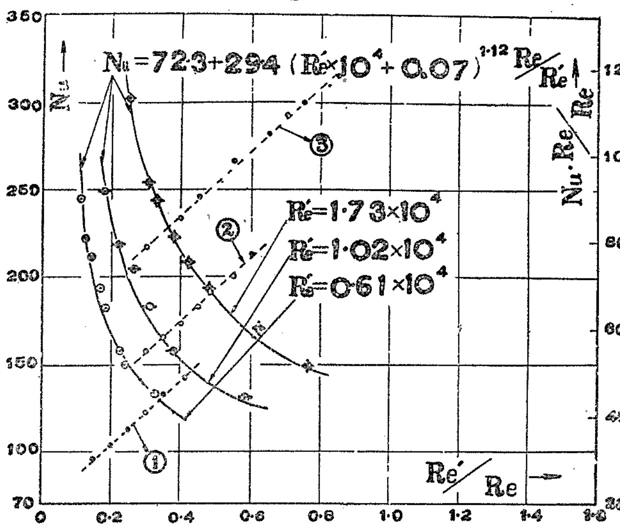
Fig. 2



In Fig. 2, the full line Nus=1.01 $\sqrt{Re}$ gives the Schmidt's theoretical results at the stagnation point of the cylinder, and the dotted line the writer's experimental ones in the case of still cylinder,

In Fig. 3, the relation between Num and Re′/Re is shown by graphically, and from the figures it is clear that the increment of Num lies in the ranges 100~250 for Re′/Re=0~1.0, where, Re represents Reynolds's number $v_\infty d/\nu$, and Re′ equivalent Reynolds's number $v_\theta d/\nu$.

F g. 3



4. Empirical formulae.

The values of Num are able to be found from the eq. (7) by graphical integration, but now if we use the asymptotic expansion for large vales of [$h\phi_R$], eq. (8) takes the form as

$$\mathrm{Num}=\frac{2h}{\pi^2}\oint\sqrt{\frac{\pi}{2h\phi_R}}\left[1+\frac{3}{8h\phi_R}-\frac{1}{128h^2\phi_R^2}+,-,\cdots\cdots\right]d\phi_R \tag{9}$$

In this equation, for the non-circulating case

$$\mathrm{Numo}=\mathrm{Const.}\ \sqrt{\mathrm{Pr}}\ \ \sqrt{\mathrm{Re}} \tag{10}$$

is hold and becomes Const. $\sqrt{\mathrm{Pr}}=1.14$ by calculation. As the experimental results are represented by the following equation

$$\mathrm{Numo}=0\cdot 96\sqrt{\mathrm{Re}} \qquad (11),$$

the theoretical values of Numo are larger about 20% than the experimental ones. By re-writing the experimental values in Fig.3 with co-ordinate of Num. Re′/Re ∼Re′/Re on the same graphs, we obtain the new points represented by 1,2 and 3 respectively.

Now, assuming these points lie on the lines

$$\mathrm{Num}=b+a\mathrm{Re}/\mathrm{Re}' \qquad (12),$$

we can deduce a, b from the experimental values. First, the value of b nearly takes $b=72.3\pm0.4$ from the above data, because these three lines have the same inclination.

Next, we introduce the eq. (10) to determine the value of a, and let c, d and n be the unknown coefficients, then equation referring to a takes the form

$$a=c(\mathrm{Re}'+d)n,$$

then, after simple calculations we have for the circulating flow

$$\mathrm{Num}=72.3+29.4\ (\mathrm{Re}'.10^4+0,077)^{4\cdot12}.\mathrm{Re}/\mathrm{Re}' \qquad (13),$$

The numerical results of eq. (13) are shown by three smooth curves in Fig.3. In the above analysis, we must pay attention to the fact that these results are not corrected for the radiation effects, namely, $\alpha_m=\alpha_{rad.}+\alpha_{conv.}$, and not suitable the cases in free convection.

5. Conclusion.

Comparing the results given from the wind tunnel tests with those of the theoretical analysis for the equation of temperature distribution treated as a circulating potential flow, the writer succeeded in finding the empirical formulae (12) and (13).

The experimental results and formulated curves are shown in Figs. 2 and 3. From these figures it is clear that the mean Nusselt's number Num are not only depend on Reynolds's number Re, but also on equivalent Reynolds's number Re′ for rotating cylinder; while, for a non-circulating flow the values of Numo depend only on $\sqrt{\mathrm{Pr.}}\sqrt{\mathrm{Re.}}$ As the experiments have been performed in the narrow range, that is, $v_\infty=2\sim 18$ m/s, and $v_\theta' =1.8\sim5.3$m/s, eq. (12) and (13) hold good only for these ranges and cannot be applied further.

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回轉せる無限円壔よりの熱傳達について

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摘 要

回轉している物体よりの熱傳達に関する研究の一部として，無限円壔の場合をとりあげ風洞実験による結果を簡單なポテンシアル流の理論と比較して実験式を作成した。回轉しない場合のヌツセルト数は Schmidt が岐点における値として $Nus=1.01\sqrt{Re}$ を与えているのに対し，筆者の平均ヌツセルト数は (11) 式であらわされた。また回轉しているときのそれは (13) 式であらわされ，いづれも図 2,3に示されるような結果となつた。

以上の実験は風速．回轉速度はいづれも狹い範囲で行はれたので一般性は少い。廣範囲の実験および有限円壔，円板の場合の実験は現在準備中である。実験式については昭和26年4月3日日本機械学会東京総会で講演したものであることを附記する。